

Interplay of dc current and microwaves in the magnetotransport of two-dimensional electron systems

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(Received 31 May 2009; revised manuscript received 22 September 2009; published 3 November 2009)

We theoretically study magnetoresistance oscillations in two-dimensional electron systems in the presence of microwaves and a dc electric field. We obtain that the microwave-induced resistance oscillations and zero resistance states are dramatically affected by a dc electric field of increasing intensity. The interplay of both fields produces a plasma wave which oscillates at the frequency of microwaves but with a phase difference of π radians. This plasma wave interferes with the microwave-induced electronic motion changing gradually the resistance oscillations profile: maxima evolve to minima and vice versa. We introduced in our model anharmonicity corrections to magnetoresistance oscillations in order to explain the peculiar biased profile that experimental results present. The theoretical outcomes are in agreement with experimental evidence.

DOI: [10.1103/PhysRevB.80.193302](https://doi.org/10.1103/PhysRevB.80.193302)

PACS number(s): 73.40.-c, 73.50.-h, 78.67.-n

The response of transport properties of nanodevices to radiation is one of the most active fields of research in the condensed-matter community¹ both from basic and application perspectives. From the basic standpoint, microwave (MW)-induced resistance oscillations (MIROs) and zero resistance states (ZRSs) in two-dimensional electron systems (2DESs) subjected to a perpendicular magnetic field (B) are important properties recently discovered.²⁻⁴ Intense theoretical⁵⁻¹⁴ efforts have been devoted to the study of such striking effects in order to unveil their physical origin. However the mechanism responsible is still under debate. Another remarkable effect has been observed when a 2DES under a perpendicular B is subjected to a high intensity dc electric field (E_{dc}) which produces an intense current (I_{dc}) in the transport direction.¹⁵⁻¹⁷ These experiments report oscillations in the longitudinal magnetoresistance (R_{xx}) that are periodic in $1/B$, as MIRO. Different theoretical alternatives have been already proposed to explain these dc-induced oscillations.^{18,19} Over the past few years another set of experimental evidences have been reported showing the combined effect of MW and an intense I_{dc} .^{20,21} According to them the presence of an increasing I_{dc} through the device affects MIRO in a nontrivial way: photoresistance maxima evolve into minima and vice versa. Also, MW-induced ZRS are destroyed when the current intensity is high enough. These experimental outcomes reflect a strong coupling and interplay of MW and E_{dc} effects. Some theoretical models have been already presented to explain this striking results. We can highlight the work by Khodas *et al.*,²² with an approach based in the quantum kinetic theory and the contribution of Lei *et al.*²³

In this Brief Report we present an alternative theoretical approach to address those results. We have developed a common extension of two previous theoretical models, MW-driven electron orbits⁵ and E_{dc} -field-induced plasma wave excitation,¹⁸ to finally obtain a microscopical model. According to the MW model, when a 2DES subjected to B is illuminated with MW, electron orbits are forced to move back and forth oscillating harmonically at the frequency of MW (ω). Then, the electron orbit center position is displaced by x_{cl} that is given by

$$x_{cl}(t) = A \cos \omega t \quad (1)$$

$$= \frac{eE_0}{m^* \sqrt{(w_c^2 - \omega^2)^2 + \gamma^4}} \cos \omega t, \quad (2)$$

where e is the electron charge, w_c is the cyclotron frequency, E_0 is the MW electric field, and γ is a sample-dependent damping parameter which affects dramatically the driven electronic orbits motion. Along with this movement there occur interactions between electrons and lattice ions yielding acoustic phonons and producing a damping effect in the electronic motion.⁵ On the other hand, following the E_{dc} model,¹⁸ if a 2DES subjected to B is also subjected to a E_{dc} of increasing intensity, electron orbits displace their center a distance X proportional to E_{dc} . This distance, in the same direction as E_{dc} , is given by $X = eE_{dc}/m^*w_c^2$. The displacement of the electron orbit centers with respect of positive lattice ions disturbs the whole 2D electron gas which departs from equilibrium giving rise to two lines of opposite charge at every end of the sample [see Fig. 1(a)]. Thus, an electric field E_p is built up in the sample which reads¹⁸ as

$$E_p = \frac{n_e e X}{2\pi \epsilon L}, \quad (3)$$

where L is the sample length and n_e is the 2D electron density. E_p is proportional to E_{dc} being related by $E_p = (w_p/w_c)^2 E_{dc}$. E_p tends to restore the system to its equilibrium position producing in the system a collective excitation or plasma wave. Eventually the 2D electron gas obeys the equation of motion of a harmonic oscillator^{18,24} that leads to oscillations at the two-dimensional plasma frequency w_p :

$$w_p = \sqrt{\frac{n_e e^2}{2\pi m^* \epsilon L}}. \quad (4)$$

When the two fields are present, the MW-driven oscillating orbit motion, together with the E_{dc} -induced orbit center displacement, produce an alternating change in position of the opposite charged stripes at every end of the sample [see Figs. 1(b) and 1(c)]. Therefore, E_p changes its direction during the

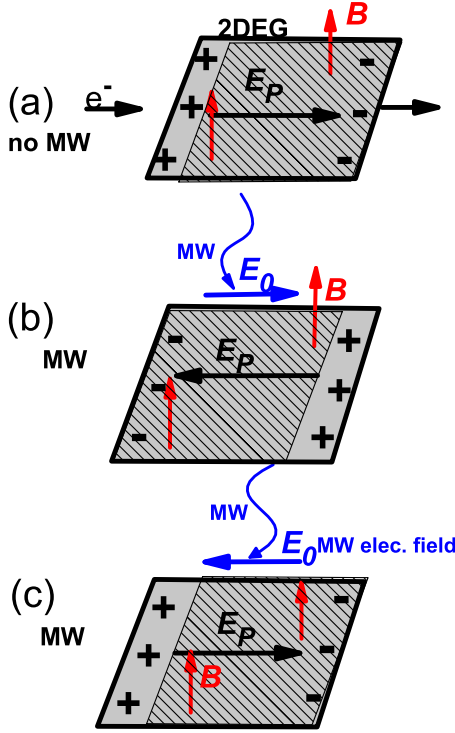


FIG. 1. (Color online) Schematic diagrams showing the dynamics of a two-dimensional electron system driven by a MW field and a dc electric field. (a) dc excitation with no MW. [(b) and (c)] Joint effect of a dc field and MW at two opposite positions of the MW-driven 2D electron gas oscillation. The MW-driven oscillating orbit motion, together with the E_{dc} -induced orbit center displacement, produce an alternating change in position of the charged stripes at every end of the sample. As a result, the built-up electric field E_p changes its direction during the MW-driven oscillation becoming oscillatory at the same frequency ω . Yet, E_p and the MW electric field, sharing equal frequency, do not oscillate in phase. This is because E_p is always opposite to MW-driven oscillating effect on the electron orbits, resulting in a phase difference of π radians. Thus, they oscillate in antiphase.

MW-driven oscillation becoming oscillatory at the same frequency ω . Importantly, although E_p shares the same frequency as the electric field of MW, they do not oscillate in phase. This is because E_p is always opposite to MW-driven oscillating effect on the electron orbits, resulting in a phase difference of π radians [see Figs. 1(b) and 1(c)]. Then, E_p plays the role of a harmonic excitation force on the 2DES giving rise to a plasma wave which oscillates also at ω and a phase difference of π radians with respect to MW. We can express this excitation force as

$$F_p = eE_p \cos(\omega t + \pi). \quad (5)$$

Then, the plasma equation of motion for the whole electron system can be written as^{18,24}

$$Nm^* \frac{d^2x}{dt} + Nm^* \gamma \frac{dx}{dt} + Nm^* \omega_p^2 x = NF_p, \quad (6)$$

where N is the number of quantum oscillators, i.e., electrons and γ is a damping factor due to interaction with the lattices

yielding acoustic phonons. Solving this differential equation we obtain the electron orbit displacement due to the E_p -driven plasma wave:^{18,24}

$$x_p(t) = A_p \cos(\omega t + \pi) \quad (7)$$

$$= \frac{eE_p}{m^* \sqrt{(\omega_p^2 - \omega^2)^2 + \gamma^4}} \cos(\omega t + \pi). \quad (8)$$

The final effect is that the excited plasma wave interferes with the MW-driven 2DEG, having both oscillating motions the same frequency ω but a relative phase difference of π radians. Thus, they oscillate in antiphase and the interference effect is subtracting. Then, if the amplitudes E_0 and E_p are similar, both types of oscillations compensate each other and MIRO and ZRS disappears. On the other hand, if E_0 is smaller than E_p , E_p -induced oscillations prevail and MIRO are inverted, i.e., maxima become minima and vice versa without changing their relative positions.

To reflect these effects in the transport properties, we apply a semiclassical model²⁵ to calculate the dissipative current, conductivity, and finally the resistivity. The drift velocity and scattering rate of the electron motion in the x direction is calculated with a quantum mechanical treatment. First, we introduce the scattering suffered by the electrons due to charged impurities^{5,18,25} considering that now the electron orbit center coordinates is written as $X^T = X^0 + x_{cl} + x_p$, where X^0 is the static orbit center position. Applying Fermi's golden rule, we calculate the electron-charged impurity scattering rate $W = 1/\tau$, where τ is the scattering time. Second we find the average effective distance advanced by the electron in every scattering jump:

$$\Delta X^{MW} = \Delta X^0 + A \cos \omega \tau + A_p \cos(\omega \tau + \pi). \quad (9)$$

If the average value ΔX^{MW} is different from zero over all scattering processes, the electron possesses an average drift velocity v in the transport directions. This drift velocity can be expressed in function of the scattering rate as $v = \Delta X^{MW}/\tau$. This drift velocity can be introduced in the classical expression of the current and finally the longitudinal conductivity σ_{xx} can be calculated:

$$\sigma_{xx} = \frac{2e}{E_{dc}} \int \rho(E) \frac{\Delta X^{MW}}{\tau} (f_i - f_f) dE \quad (10)$$

being f_i and f_f the corresponding distribution functions for the initial and final Landau states, respectively, $\rho(E)$ is the density of Landau states, and E is the energy. To obtain R_{xx} we use the relation $R_{xx} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2) \approx \sigma_{xx}/\sigma_{xy}^2$, where $\sigma_{xy} \approx n_i e/B$ and $\sigma_{xx} \ll \sigma_{xy}$. Therefore, R_{xx} is proportional to the amplitudes A and A_p :

$$R_{xx} \propto A \cos \omega \tau + A_p \cos(\omega \tau + \pi). \quad (11)$$

In Fig. 2, we present calculated R_{xx} as a function of B under MW and dc excitation. MW frequency is $\omega = 2\pi \times 69$ GHz. Direct current I_{dc} runs from 0 μA (black curve) to 100 μA (light blue curve) in intervals of 5 μA . It can be observed that as I_{dc} (E_{dc}) increases, MIRO are gradually reduced and

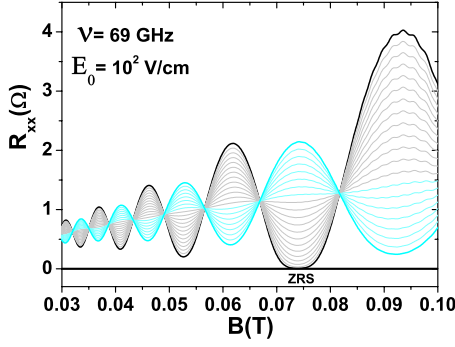


FIG. 2. (Color online) Calculated R_{xx} versus B of a 2DES under MW and E_{dc} excitation. The current I_{dc} runs from 0 μA (black curve) to 100 μA (light blue curve) in 5 μA intervals. As I_{dc} (E_{dc}) increases, the subtracting interference becomes more intense and MIRO are gradually reduced and ZRS are destroyed. For high enough I_{dc} , the whole profile of R_{xx} is inverted: maxima (minima) evolve into minima (maxima). $T=1$ K.

ZRS are destroyed. For high enough I_{dc} , the whole profile of R_{xx} is inverted: maxima (minima) evolve into minima (maxima). These calculated results are in reasonable agreement with experiments.²¹

With the present theoretical model we obtain the main experimental features, however we do not recover the peculiar *distorted* profile that MIRO present in the experimental results.²¹ To recover this striking profile we have introduced in our model anharmonic corrections for x_{cl} and x_p following Ref. 24. According to the experimental parameters used,²¹ experiments were carried out at full MW power. This implies that the amplitude of the MW-driven orbit oscillations becomes very large giving rise eventually to anharmonic behavior. In other words, under this regime, the whole 2DES performs MW-induced anharmonic oscillations.²⁶ As a result, the E_p -induced plasma wave becomes anharmonic as well mirroring the MW-induced electronic motion but with π radians of delay. Since we do not know the exact nature of the anharmonic term in the corresponding potential, it is impossible to solve analytically the classical equations of motion.²⁷ However we can take an alternative approach if we consider that although not harmonic, the system can be considered still periodic. As for any periodic function, we can try to express x_{cl} and x_p through a Fourier series and propose a solution like

$$x_{cl}(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(nwt) + B_n \sin(nwt)], \quad (12)$$

$$x_p(t) = \frac{A'_0}{2} + \sum_{n=1}^{\infty} [A'_n \cos(nwt + \pi) + B'_n \sin(nwt + \pi)], \quad (13)$$

where A_0 , A_n , B_n , A'_0 , A'_n , and B'_n are the corresponding Fourier coefficients. Then, under a regime of anharmonicity the average effective distance advanced by the electron in every scattering jump is given by

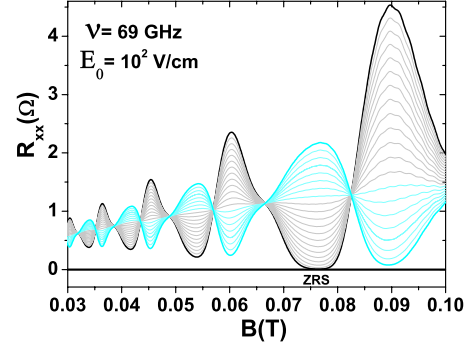


FIG. 3. (Color online) Same as Fig. 2 with anharmonic terms included in R_{xx} . The anharmonicity is reflected in the distorted profile of R_{xx} which is in reasonable agreement with experiment. The inclusion of anharmonic corrections are reflected in the distorted profile of MIRO recovering the experimental biased profile. $T=1$ K.

$$\begin{aligned} \Delta X^{MW} \propto & \sum_{n=1}^{\infty} [A_n \cos(nw\tau) + B_n \sin(nw\tau)] \\ & + \sum_{n=1}^{\infty} [A'_n \cos(nwt + \pi) + B'_n \sin(nwt + \pi)]. \end{aligned} \quad (14)$$

In order to obtain the Fourier terms, we have carried out a Fourier synthesis process. This process consists in constructing the ΔX^{MW} form by adding together a fundamental frequency (which corresponds to the harmonic case) and overtones of different amplitudes keeping the number of Fourier terms as small as possible. Since at this stage it is impossible to obtain analytical expressions for the Fourier coefficients, we have introduced phenomenologically the coefficients A_n , B_n , A'_n , and B'_n . Thus for instance for A_n and B_n we have used the following expressions:

$$A_n = \alpha_n \frac{eE_0}{m^* \sqrt{[w_c^2 - (nw)^2]^2 + \gamma^4}} \quad (15)$$

and

$$B_n = \beta_n \frac{eE_0}{m^* \sqrt{[w_c^2 - w^2]^2 + \gamma^4}}, \quad (16)$$

where α_n and β_n are anharmonicity terms. Their values are getting larger as the anharmonicity increases. Similar but not identical expressions have been used for A'_n and B'_n . Finally we can express R_{xx} as being proportional to a sum of Fourier terms which come from x_{cl} and x_p ;

$$\begin{aligned} R_{xx} \propto & \sum_{n=1}^{\infty} [A_n \cos(nw\tau) + B_n \sin(nw\tau)] \\ & + \sum_{n=1}^{\infty} [A'_n \cos(nwt + \pi) + B'_n \sin(nwt + \pi)]. \end{aligned} \quad (17)$$

In Fig. 3, we present calculated R_{xx} versus B under MW and dc excitation including the anharmonic terms. MW and I_{dc}

parameters are the ones of Fig. 2. The inclusion of anharmonic corrections are reflected in the distorted profile of MIRO recovering the peculiar experimental shape. These figures illustrate how the R_{xx} maxima evolve to minima and vice versa as the current intensity is progressively increased. It can be observed clearly the anharmonicity feature of distorted profile in the R_{xx} oscillations. For big enough MW power other anharmonicity features would be observable like, for instance, new resonance peaks at the subharmonics of the cyclotron frequencies. All these features correspond unambiguously to a slightly anharmonic behavior. However when a nonlinear system is driven with very large amplitude, new vibrational phenomena appear, such as vibrations in which the motion only repeat itself after two or more driver periods leading the systems finally into a chaotic regime.²⁸ This latter case is not consider in this Brief Report.

In summary, we have presented a theoretical model on the

joint effect of MW radiation and an intense dc electric field on the transport properties of a 2DES. MIRO and ZRS results are importantly affected. ZRS are destroyed and MIRO are quenched. Yet, for a high enough dc intensity MIRO are inverted and maxima evolve to minima and vice versa. According to our model a high intensity dc electric field in the direction of transport gives rise to a plasma wave which interferes with the MW-induced electronic orbit motion. The dc-excited plasma wave has a relative delay of π radians with respect to the latter producing a subtracting interference effect. Then, MIRO are initially quenched and eventually peaks and valleys change their relative positions. We have introduced anharmonic effects in our model in order to explain the biased profile that experimental MIRO present.

This work was supported by the MCYT (Spain) under Grant No. MAT2008-02626/NAN.

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